Conn., was open to the end of the month, the latest date since 1900.

SNOW.

Following is a very brief *résumé* of the snow bulletins issued in the various western States where the run-off from the melted snows is depended upon to supply water for irrigation purposes.

Arizona.—At the end of December there were rather less than 3 inches of snow in the mountains tributary to the Salt, Verde, Agua Fria, Hassayampa, and Little Colorado rivers, with none in the valleys. It is estimated that the run-off will last until March, 1908.

Colorado.—There was rather less than the usual amount of snow in the Rio Grande and other southern watersheds, and about the average amount over the northern watersheds. Later snows must be depended upon to furnish the water supply for irrigation purposes.

Idaho.—The amount of snowfall was somewhat above the average, altho of uneven distribution. As there was also considerable rain, the accumulated snow has become well solidified, insuring a high percentage of run-off.

Montana.—The snowfall was deficient, and at this time prospects of an abundant water supply are not favorable. This is in marked contrast to the conditions that existed during December, 1906, when there were several inches of well-packed snow on the ground.

Nevada.—The same conditions prevailed as in Montana. In the Humboldt Basin there were about 2 feet, and in the Truckee Basin about 5 feet of snow near the mountain summits. It is too early to make an accurate estimate of the water supply that will be available later, but more snow will be necessary for even a normal amount.

New Mexico.—The snowfall was comparatively light, but conditions, on the whole, are favorable for future water supply. The depth of snow in the mountains varies from 7 to 19 inches, with the maximum amount over the Rio Grande watershed.

Oregon.—The snowfall was deficient in quantity except over the higher altitudes in the extreme eastern portion of the State, where the depth is considerably greater than at the end of the year 1906.

Utah.—The snowfall was of average quantity, and the prospects of a sufficient water supply are favorable.

Washington.—The snowfall was somewhat deficient in quantity, but is quite compact owing to abundant rains and high temperatures.

Wyoming.—There is an abundant, compact snowfall as a rule, insuring, under normal conditions, a good supply of water for the coming summer, except over the eastern slope of the Big Horn Mountains.

The highest and lowest water, mean stage, and monthly range at 199 river stations are given in Table VI. Hydrographs for typical points on seven principal rivers are shown on Chart I. The stations selected for charting are Keokuk, St. Louis, Memphis, Vicksburg, and New Orleans, on the Mississippi; Cincinnati and Cairo, on the Ohio; Nashville, on the Cumberland; Johnsonville, on the Tennessee; Kansas City, on the Missouri; Little Rock, on the Arkansas; and Shreveport, on the Red.—*H. C. Frankenfield, Professor of Meteorology.*

SPECIAL ARTICLES, NOTES, AND EXTRACTS.

COMPREHENSIVE MAPS AND MODELS OF THE GLOBE FOR SPECIAL METEOROLOGICAL STUDIES.[1]

By Prof. CLEVELAND ABBE.

1. Maps in general.—Comprehensive maps of the globe for special meteorological work are needed in studying the general motions of the atmosphere as preparatory to long-range forecasts. Maps or charts that shall represent a large portion of the earth's surface are described in many treatises on chartography such as those of Gretschel, Schott, Craig, and others. Each method of charting results in a plane figure that represents some one feature of the spherical surface correctly, but is necessarily distorted as to other features.

2. Geometrical projections.—If a spherical surface is viewed from one fixt point and is thus directly projected on the retina or on a plane, we obtain the geometrical perspective methods of projection such as those known as the orthographic, the stereographic, or the gnomonic, all of which were known to the ancients, or the newer projections devised by James, Clarke, and others.

3. Geometric developments.—If the spherical surface is projected perspectively on the surface of a cylinder or cone tangent to some small circle of the sphere, or intersecting the sphere, or even wholly outside of the sphere, and if this surface be then unrolled or "developed" on a plane surface, we obtain a geometric development of the sphere as distinguished from a geometric projection, properly so called. Such are Mercator's projection, the conic, the polyconic, and Bonné's.

4. Analytical developments.—If special conditions are imposed as to the relative dimensions or distortions in different parts of the desired map, then the transfer from sphere to plane must in general be done by the help of analytical formulas and by computations rather than by the so-called geometric constructions; these we call analytical developments, as, for example, the equal-surface development of Lambert, or the minimum-distortion method of Gauss, or the "balance of errors" method by Airy.

5. Special cases.—Finally, there are some other methods of drawing maps, described at first as purely arbitrary methods, but eventually shown to be special cases of the methods previously mentioned. Such are Mercator's, the globular method of Nicolosi, and the polar projection described by both Postel and Werner independently, which has been used, for example, by the Weather Bureau since 1875 in the daily weather maps of the Daily Bulletin of International Simultaneous Observations, as also in the summary of these observations published in Bulletin A of the Weather Bureau.

6. Angular relations.—Maps like the Mercator and gnomonic, in which the angular relations on the spherical surface remain unchanged in the resulting maps, are specially favorable for plotting local wind directions and for studying the angles between winds and isobars.

7. Equal areas.—Maps that preserve a uniform ratio between all areas on the globe and the corresponding areas on the maps are the so-called equal-surface developments, and are appropriate for the study of areal statistics, such as the distribution of rainfall, the extent of high and low areas, the mass of air or moisture or evaporation, the average pressures or temperatures in special zones of the globe, the relations between insolation and temperature, the relative frequency of local storms over a given area, etc.

8. Equal distances.—Maps in which equal elementary distances on the sphere, both meridional and latitudinal, have corresponding equal equivalents on the map, are generally most appropriate for measuring the lengths of storm tracks, the movement of the wind in twenty-four hours, the intensity of the gradients of temperature and pressure. These include the polyconic projections, but for long distances the angular distortions are appreciable and the gnomonic projection must be used.

9. Value of polar projections in meteorology.—All methods of projection or development are of general application to the

[1] Read before the Association of American Geographers at its meeting in New York, N. Y., January 1, 1907. Revised February 5, 1908. The figures accompanying this article appear upon Plates I, II, III, and IV, at the end of this issue.

earth's surface, without regard to the position of the pole or the equator; but in some studies it is best to make the equator, or some special meridian, or some special small circle, prominent in the map, while in other cases it is necessary to make the North or South Pole the prominent feature. For special studies in dynamical meteorology, including the relation of solar radiation to the movements of the atmosphere, the polar projections of the whole of the Northern and Southern hemispheres are necessary and peculiarly convenient. For such purposes we may use the orthographic, the stereographic, the gnomonic, the arbitrary Postel-Werner, the equivalent or equal-area (of Lambert), the James, the Airy, or the Clarke projection. The special merits of each of these projections, when applied to polar maps, will be considered in detail in the following paragraphs.

The general graphic comparison of the construction and relative dimensions of various polar maps is given by the study of Plate II, where NES is a section of the surface of a hemisphere whose center is O and polar axis NOS. The Northern Hemisphere NE is projected on the plane that is tangent to the sphere at the North Pole. The equator at E and small circles of latitude as at L will be projected as concentric circles on this plane of projection, and the meridians will become straight lines passing thru the North Pole N and its corresponding projection n. The several methods of construction will be considered in the following paragraphs.

10. *Perspective projection: gnomonic.*—When the eye is at the center O and any radius OL is prolonged this intersects the tangent plane at a point l_g in that plane as the representative of the original point on the sphere. All great circles on the globe lie in planes passing thru its center, and are therefore represented by straight lines on the map drawn on our tangent plane in this gnomonic projection. The angles between the meridians of the globe are the same as the angles between the corresponding meridional lines on the map. As we depart from the pole N the radii of the projected circles that represent small circles of latitude become grossly exaggerated (thus AL becomes nl_g) and the radius becomes infinite for the equatorial circle, since the line OE is parallel to the tangent plane. These radii in general are computed by the formula,

$$\rho_g = nl_g = R \tan \theta = R \tan (90^\circ - \beta) = R \text{ cotan } \beta,$$

where R is the radius of the sphere, θ the north polar distance, β is the latitude, and ρ_g is the radius on the gnomonic map corresponding to any given small circle of latitude (β) on the sphere.

Charts have been published by the Hydrographic Office of the United States Navy so as to make this gnomonic projection available for the use of the navigator in drawing the shortest possible (or "great circle") route between any two points on the ocean.

11. *Perspective projection: stereographic.*—When the eye is placed at the South Pole S on the surface of the sphere, diametrically opposite to N, the perspective lines SE, SL, etc., will meet the tangent plane in the points e_s l_s, and the corresponding radii for the map nl_s are computed by the formula,

$$\rho_s = nl_s = 2\ R \tan \tfrac{1}{2}\ \theta = 2\ R \tan \tfrac{1}{2}(90^\circ - \beta)$$

where the letters have the same meanings as before.

In this projection all meridians on the sphere become straight lines passing thru n on the map or the tangent plane. Great circles that do not pass thru N, but are inclined to the axis of the earth by any angle, become circles on the plane of projection and intersect the equator at points diametrically opposite to each other.

The projection of the Northern Hemisphere NE as seen from S may be extended to include a large part of the Southern Hemisphere, but the distortion soon becomes excessive; however, altho not used for maps, such extension may be very convenient for the study of special geometrical problems.

All small circles drawn anywhere on the sphere are projected as circles on the plane. The practical methods of study elaborated by the late Prof. S. L. Penfield, of Yale,[2] make these stereographic projections a most convenient method for working problems in spherical trigonometry, and maps of hemispheres on this projection are useful in many meteorological problems.

12. *Perspective projection: orthographic.*—If the eye is placed beyond S, on NOS prolonged and at an infinite distance from O, the lines of sight become parallel to NOS and normal to the tangent plane nt. The point l_0 is the orthographic projection of L as seen from this infinite distance. The radius of the globe OE is not foreshortened for points on the equator, but the radius as projected for any other point, whose latitude is β, is foreshortened and becomes:

$$nl_0 = \rho_0 = R \sin \theta = R \cos \beta.$$

Small circles of latitude are projected as circles, but all other small circles on the globe become ovals on the map. Small circles parallel to a meridional circle become straight lines parallel to it. If the center of an inclined small circle is at the north polar distance θ, and if the angular radius of the small circle is α, then on the globe the linear radius is $R \sin \alpha$, but on the map the longest diameter of the projected oval is perpendicular to its central meridian and is $2R \sin \alpha$, while the shortest diameter of the oval is in the direction of the central meridian and is $2R \sin \alpha \cos \theta$.

The moment of inertia of any portion of the atmosphere is proportional to the square of its distance from the earth's axis of rotation, which distance is the same as ρ_0, therefore a map on this orthographic projection, when rotating about n, gives correct ideas as to moments of inertia and also as to linear distances traversed by any point in its diurnal rotation. This projection is therefore convenient for studying mechanically those problems of rotation and inertia that are treated analytically by Helmholtz and Brillouin. In so far as any other projection increases the radius it distorts the moment of inertia by a quantity that may easily be calculated.

13. *Equal-surface development.*—In general the maps that correspond to this title are not geometrical projections, but must be prepared by the help of numerical tables based on formulas to be deduced by the aid of the differential and integral calculus. In this development areas on the sphere are represented by proportionate areas on the map, or there is no distortion of areas; so that for every point on the globe the differential of the area of the spherical surface, or,

$$dA = 2\pi R^2 \sin \theta\ d\theta,$$

must be equal to the differential of the corresponding area on the map, which latter is $da = 4\pi\rho\ d\rho$.

In the special case of a polar projection each zone of latitude on the sphere must be represented on the map by a circular ring of equal area, and this is realized when the point L on the sphere is transferred to the point l_q on the map by using the corresponding chord of the sphere, or NL, as its radius. If the north polar distance NOL is θ, as before, then the length of this chord is $\rho_q = 2R \sin \frac{1}{2}\ \theta$ and the area of its circle is $\pi\rho_q^2 = 4\pi R^2 \sin^2 \frac{1}{2}\theta = 2\pi R^2 (1 - \cos \theta)$.

The area of any spherical zone between two latitudes is

$$A = 2\,\pi\,R^2 (\cos \theta - \cos \theta'),$$

while the area of the corresponding ring on the map is

$$a = 2\,\pi\,(\rho^2 - \rho'^2).$$

This form of development was first given by Lambert in 1719. A modification of it, known as Mollweide's, is applicable to equatorial projections and other forms of charts than the polar projection.

DeLorgna's polar map is a similar development in which each parallel becomes a circle whose radius is a mean pro-

[2] See Am. Jour. Sci., January, 1901 (4), XI.

portional between the diameter of the sphere and the height of the spherical segment belonging to that parallel.

14. Equi-meridional polar development: Postel-Werner.—In this map the curvilinear distance from the North Pole to any point, L, on the earth's surface is laid off on the map from n to l_p, therefore nl_p is the arc of which NL is the chord; it is therefore a little longer than NL, consequently any circle of latitude on this development lies a little beyond the corresponding circle in the equal-surface development. All meridians and angular rotations are correct and without distortion; all meridional distances are correct, but other distances and other angles are more or less distorted. The areas are very appreciably distorted, as also the distances at right angles to the meridian, when we get beyond 60° from the North Pole. This development was devised independently by Postel and Werner and was adopted by the Signal Service in the preparation of maps for the Daily Bulletin of International Simultaneous Meteorological Observations.

15. James's method.—The methods of charting devised respectively by Sir Henry James and Sir George B. Airy may have some special advantages for meteorological work.

In the proceedings of the Royal Geographical Society, London, 1857,[3] Sir Henry James states that some remarks by Sir John Herschel and Sir Charles Lyell led him to devise a projection that would include in one map as large a part of the earth's surface as in any way practicable by methods of projection. James's projection is a perspective method, properly so called, and may be thus stated. From a point of vision, V_j, (Plate II) outside of the sphere draw a tangent cone V_jQ; prolong this cone until it intersects the tangent plane at the point q_j and project the whole spherical surface NEQ on to this tangent plane. The angle NOQ will be larger than 90°. The region near N will be mapped with but little distortion; the region near Q will be greatly distorted. In De la Hire's globular projection of 1704 the visual point is so placed that VO is 1.707 times the radius; in Lowry's projection it is 1.69 times the radius. In Parent's projection of 1702 the visual point is placed at 2.105 times the radius, and in his second projection of 1713 it is placed at 1.594 times the radius, but James made the distance of the visual point from the center still smaller, namely, 1.5 times the radius, and therefore 2.5 times the radius from the tangent point N.

The angle NOQ, or the polar distance at the edge of the possible map, is 138° 12′; but on account of the distortion near the edge, James extended his map of 1857 only to 113° 30′; that is to say, if the center of his map had been the North Pole, it would have stopt at the Tropic of Capricorn; but by placing the center of his map at a point in latitude 23° 30′ N. and longitude 15° E., he was able to present in one chart nearly the whole of the continental portions of the globe. Such a general view of the earth is exceedingly useful in many geophysical studies, such as tides, luni-tidal strains, earthquakes, and terrestrial magnetism, and is very instructive in the study of commercial statistics and history.

16. Airy's and Clarke's methods.—Following the publication of this memoir by James came a memoir by Airy published in the London, Edinburgh, and Dublin Philosophical Magazine, 1861,[4] describing a form of development in accordance with the principle of the "balance of errors" as Airy calls it. This projection agrees with that of James as to general appearance of the resulting map, but differs as to the basic principle. A slip in Airy's analysis was corrected by A. R. Clarke and communicated by James to the Philosophical Magazine, 1862.[5] Airy's method is sometimes spoken of as a modification of James's, but it is an entirely independent method—it is a development and not a geometric projection, as he deals with the general problem of the development of a spherical surface on a plane. In this same communication of 1862, James also publishes a new projection by Clarke, who applied Airy's "balance of errors" to the problems of projection properly so called. Clarke's projection of 1862, is, therefore, a geometrical or visual projection, subject not only to the conditions peculiar to projections, but also to the conditions implied in the principle of the "balance of errors." Clarke's geometrical projection imposes a slight restraint on the freedom of Airy's development, and the results of the two methods are not mathematically identical, but are so nearly so that for ordinary maps of a hemisphere the differences are very small.

Clarke's method is graphical and can be used by any draftsman, but Airy's is analytical and requires computations and numerical tables. Neither of these two can properly be spoken of as a modification of James's projection, they having merely grown out of his first memoir, historically, by a process of suggestion. It so happens that Clarke's projection for the special case of a hemisphere becomes identical with the central part of James's projection, where the visual point is placed at the distance 1.5 R from the center; but this must be regarded as an accidental item, since Clarke's projection for the whole polar distance of 138° 12′, that might have been included in James's map, differs appreciably in its exterior portion from that map, and is much better than it in the matter of distortions.

In Airy's development nothing is said about the location of a visual point or the location of a plane of projection relative to the surface of a sphere. These matters are left entirely out of consideration, and the problem is one of pure analysis. In Clarke's memoir the location of the visual point and the place of projection relative to the sphere are specially considered, and each is made movable, so that by proper adjustment and combination the relation between them becomes such as to produce by projection a map that approximately satisfies the idea of a "balance of errors."

17. Airy's development.—Both projections and developments alike alter the relative proportions of the spherical distances, areas, and angles. The distortions thus introduced, by reason of which a map differs from the original sphere, are very undesirable, but inevitable, and force us in studying any problem to select the projection specially adapted to the question in hand.

Airy introduced the idea that the errors of distortion should be treated as errors of observation are treated in Gauss's method of least squares. He sought for a map satisfying the one condition that on the average over the whole surface of the map the square of the distortion in area plus the square of the distortion in figure or shape shall be a minimum. According to this idea one must first determine how much of the sphere is to be covered by the map, and then make the location of each point such that the sum of the squares of all the distortions shall be a minimum for that particular map. Airy called this the "balance of errors."

This condition is shown to be equivalent to the condition exprest in the formula

$$\left(\frac{\Delta z}{z}\right)^2+\left(\frac{\Delta b}{b}\right)^2=\text{minimum},$$

where the linear distortions (Δz) in latitude and (Δb) in longitude at any point are divided by the actual latitude and longitude of that point, and the minimum relates to the whole area of the chart, which in our polar projection will be a chart whose extreme radius is that for any adopted polar distance β. For the whole Northern Hemisphere β will be 90°. Airy's solution of this problem in calculus (as corrected by Clarke) can be exprest by the four following formulas, which I have rearranged so as to make the computations as simple as possible, and in which ρ represents the linear radius on the map corresponding to any angular polar distance θ on the sphere whose radius is R:

[3] Vol. I, p. 421. [4] Vol. 22, p. 409. [5] Vol. 23, p. 306-308.

1. $C_\beta = \cot^2 \frac{1}{2}\beta$ nat log $\sec^2 \frac{1}{2}\beta$.
2. $\rho_\beta = 2\,RC_\beta\ \tan \frac{1}{2}\beta$.
3. $C_\theta = \cot^2 \frac{1}{2}\theta$ nat log $\sec^2 \frac{1}{2}\theta$.
4. $\rho_\theta = RC_\beta\left[1 + \frac{C_\theta}{C_\beta}\right]\tan \frac{1}{2}\theta$.

TABLE 1.—*A general table of C_θ, serving for the variable C_θ and the constant C_β.*

θ	Log C_θ	θ	Log C_θ	θ	Log C_θ	θ	Log C_θ
°		°		°		°	
0	0.000 000	24	9.990 386	48	9.960 141	70	9.910 485
2	9.999 932	26	.988 673	50	.956 613	72	.904 710
4	.999 668	28	.986 833	52	.952 838	74	.898 694
6	.999 409	30	.984 852	54	.948 916	76	.892 432
8	.998 936	32	.982 721	56	.944 826	78	.885 910
10	.998 350	34	.980 459	58	.940 523	80	.879 122
12	.997 621	36	.978 031	60	.936 033	82	.872 062
14	.996 748	38	.975 444	62	.931 345	84	.864 716
16	.995 712	40	.972 703	64	.926 460	86	.857 049
18	.994 602	42	.969 807	66	.921 356	88	.849 108
20	.993 364	44	.966 723	68	9.916 035	90	9.840 826
22	9.991 954	46	9.963 531				

Taking our constant, C_β, from Table 1 for any adopted limit to the proposed chart, we then compute the values of ρ_θ for each degree of angular distance from the center of the chart, which, in the present case, would correspond to each degree of north polar distance, θ. If we adopt $\beta = 90°$, or the equator, for the limit of our chart, as we shall do, we find in Table 1 the value of log $C_\beta = 9.840826$, whence $\rho_\beta = R \times 1.3863$. If we had adopted 113° 30′, as James did for the limit of his actual chart, we should have found log $C_\beta = 9.73785$ and $\rho_\beta = 1.6204$. If we had adopted 138° 12′, as was possible for James to have done with his visual point at V so that VO = 1.5 radii, we should have had log $C_\beta = 9.51255$ and $\rho_\beta = 1.6959$. With the value of $\beta = 90°$ and the corresponding C_β we compute the values of ρ for a chart of either hemisphere as given in Table 2 for every even degree of polar distance.

TABLE 2.—*Values of ρ for a map of a hemisphere on Airy's development (β=90°; log C_β=9.8408255).*

θ=polar distance.	ρ=radius.	θ=polar distance.	ρ=radius.	θ=polar distance.	ρ=radius.	θ=polar distance.	ρ=radius.
°		°		°		°	
0	0.0000	24	0.3552	48	0.7148	70	1.0551
2	.0296	26	.3850	50	.7452	72	1.0870
4	.0591	28	.4147	52	.7756	74	1.1191
6	.0887	30	.4445	54	.8062	76	1.1514
8	.1182	32	.4743	56	.8368	78	1.1840
10	.1478	34	.5042	58	.8676	80	1.2169
12	.1774	36	.5341	60	.8985	82	1.2500
14	.2069	38	.5641	62	.9295	84	1.2835
16	.2366	40	.5941	64	.9607	86	1.3173
18	.2662	42	.6242	66	0.9920	88	1.3516
20	.2959	44	.6544	68	1.0235	90	1.3863
22	.3255	46	.6847				

18. Clarke's projection by "balance of errors."—In this projection Clarke makes both the distance from the center of the sphere to V_c, the visual point, and the distance from the plane of projection, *pp*, to the center of the sphere variable or adjustable. In Plate II let $V_c p = k$ and $V_c O = h$. These are the variables whose values are to be so adjusted that the map projected on *pp*, or the computed ρ, shall correspond to the condition of the "balance of errors," as described by Airy.

The projection from V_c thru the point L on the sphere to the corresponding point, l_c, on the map, leads to the simple geometrical relation,

$$\frac{pl_c}{pV_c} = \frac{eL}{eV_c} \text{ or } \frac{\rho}{k} = \frac{R \sin\theta}{h + R\cos\theta} \text{ or } \rho = \frac{kR\sin\theta}{h + R\cos\theta}. \qquad (1)$$

Any short unit distance measured *radially* on the map is,

$$\sigma = \frac{k(1 + h\cos\theta)}{(h + \cos\theta)^2}. \qquad (2)$$

And any short unit distance measured along a latitude circle on the map is,

$$\sigma' = \frac{k}{h + \cos\theta}. \qquad (3)$$

The distortions are therefore $(\sigma - 1)$ and $(\sigma' - 1)$ and the sum of the squares, that is, the distortions of these distances squared and added together for the whole area of the map, has to be a minimum by the conditions of the "balance of errors;" that is to say,

$$\int_0^\beta \left[(\sigma - 1)^2 + (\sigma' - 1)^2\right] = \text{Minimum}. \qquad (4)$$

If we use the following notation:

$$H = \nu - (h+1) \text{ nat log } (\lambda + 1) \qquad (5)$$

$$H' = \frac{\lambda}{h+1}\left(2 - \nu + \frac{1}{3}\nu^2\right) \qquad (6)$$

$$\lambda = \frac{1 + \cos\theta}{h + \cos\theta} \qquad (7)$$

$$\nu = (h - 1)\lambda, \qquad (8)$$

then equation (4) can be written as follows:

$$M \text{ (minimum)} = 4\sin^2 \tfrac{1}{2}\theta + 2kH + k^2 H'. \qquad (9)$$

The value of this integral becomes a minimum when

$$\frac{\partial M}{\partial h} = 0 \qquad (10)$$

$$\text{and } \frac{\partial M}{\partial k} = 0, \text{ simultaneously;} \qquad (11)$$

and these conditions give us respectively,

$$0 = 2k\frac{\partial H}{\partial h} + k^2\frac{\partial H'}{\partial h} \qquad (12)$$

$$\text{and } 0 = 2H + 2k\,H'. \qquad (13)$$

This last equation (13) gives $k = -\frac{H}{H'}$, (14)

whence the fundamental equation (9) becomes,

$$M \text{ (minimum)} = 4\sin^2 \tfrac{1}{2}\beta - \frac{H^2}{H'}. \qquad (15)$$

It remains now to find values of H and H' that will satisfy the conditions of equation (15); no method of doing this directly has yet been devised, so that the operation is performed numerically and is rather tedious. With a series of assumed values of h we compute the corresponding values of λ, ν, H, H', and $\frac{H^2}{H'}$, using that value of θ, i. e., β, that belongs to the boundary of the proposed map. By examining the resulting regular series of values of $\frac{H^2}{H'}$ we easily ascertain when this latter ratio is a maximum, and consequently when the value M is a minimum; with the corresponding value of h we can then compute $k = -\frac{H}{H'}$ as in equation (14). Table 3 gives values of h and k as computed for specific values of β.

TABLE 3.—*Values of h and k corresponding to given values of β.*

β	h	k
40	1.625	2.543
54	1.61	....
90	1.47	2.034*
	1.470	2.03766†

*Clarke. †Abbe.

For meteorological charts we propose at present to use only the values for β=90°, or a hemisphere, and according to my own calculations for this polar chart the value of ρ for any value of θ is to be computed by the following formula:

$$\rho=\frac{2.03766 \sin \theta}{1.4700+\cos \theta}=\frac{[0.1418153] \sin \theta}{1+\cos \theta/[0.1673173]}$$
$$=\frac{1.386166 \sin \theta}{1+0.680274 \cos \theta} \tag{16}$$

With this formula we have computed Table 4 in order that the reader may make a detailed comparison between Clarke's projection and Airy's development for all parts of a hemisphere. The table has been extended to 95° in order to facilitate interpolations, but if the map is extended beyond 90° then the equatorial circle should be made prominent as a heavy line, since the "balance of errors" applies only to the region inside the equator.

By comparing the values of ρ in Tables 2 and 4 representing Airy's and Clarke's formulas, respectively, we see at once that they agree at the 90° limit as well as at 0°, but differ as we approach 35° where the maximum difference in the value of the radius amounts to nearly two per cent. As we have before said this difference is due to the fact that Clarke imposed upon the conditions peculiar to the "balance of errors" another geometrical consideration, namely, that the map should be a projection rather than a simple development, so that the "balance of errors" is not perfectly attained except in so far as is consistent with the geometrical projection.

TABLE 4.—*Values of ρ for $\beta=90°$, computed by Clarke's formula.*

θ	ρ	θ	ρ	θ	ρ	θ	ρ
°		°		°		°	
0	0.000000	25	0.362391	50	0.738807	75	1.138485
5	.072011	30	.436138	55	.816704	80	1.220883
10	.144140	35	.510562	60	.895770	85	1.303601
15	.216503	40	.585759	65	0 975762	90	1.386165
20	.289217	45	.661818	70	1.056707	95	1.467923

19. Numerical comparison of different projections.—Altho Plate II gives us a clear idea of the different styles of charting, yet a numerical tabular comparison is still more instructive. We propose now to compile a small table (Table 5) showing the values of the radii of the different circles of latitude for several systems of polar projections, as follows:

(1) The gnomonic projection, in which
$$\rho=R \tan \theta. \tag{17}$$

(2) The stereographic, in which
$$\rho=2 R \tan \tfrac{1}{2} \theta. \tag{18}$$

(3) The orthographic, in which
$$\rho=R \sin \theta. \tag{19}$$

(4) The equal-surface, in which the radii are chords, so that
$$\rho=2 R \sin \tfrac{1}{2} \theta. \tag{20}$$

(5) The Werner-Postel projection, in which the radii are equal to the rectified circular arcs measured from the North Pole down to any point on the sphere, so that
$$\rho=R \cdot \frac{2 \pi}{360} \cdot \theta=R \times \theta \times 0.01746. \tag{21}$$

(6) The James projection, in which the visual point is at a distance of 1.5 times the radius of the sphere, so that,
$$\rho_\theta=\frac{5/3\, R \cos \theta}{1+2/3 \cos \theta}. \tag{22}$$

(7) Airy's development by "balance of errors," in which for $\beta=90°$,
$$\rho_\theta=[9.840826] R\left(1+\frac{C_\theta}{C_\beta}\right) \operatorname{tg} \tfrac{1}{2} \theta=0.693148 R\left(1+\frac{C_\theta}{C_\beta}\right) \operatorname{tg} \tfrac{1}{2} \theta. \tag{23}$$

(8) Clarke's projection by "balance of errors," in which for $\beta=90°$,
$$\rho_\theta=\frac{2.03766\, R \sin \theta}{1.470+R \cos \theta}=\frac{1.3862\, R \sin \theta}{1+0.68027\, R \cos \theta}. \tag{24}$$

The values of ρ for a sphere whose radius is unity ($R=1$) are given in Table 5 for each 10° of north polar distance. By comparison of the numbers on any horizontal line we easily see the distortions to which the spherical surface is subjected in preparing maps on these respective projections. Thus at 80° on the gnomonic projection the meridional distance from the North Pole is represented by a distance 5.671 times the radius of the sphere, while on the orthographic projection, column 3, this distance is only 0.985 times the radius. By comparing the numbers in the last three columns it will be seen that Airy's and Clarke's methods give almost identical results, and altho they differ but little from the equal-surface projection in column 4, yet that difference is decidedly in their favor, as they contract the polar regions and expand the equatorial regions, so that the distortions in shape are appreciably diminished.

TABLE 5.—*Values of ρ for various projections in terms of R as unity.*

North polar distance.	1 Gnomonic.	2 Stereographic.	3 Orthographic.	4 Equal-surface.	5 Postel-Werner.	6 James.	7 Airy. ($\beta=90°$.)	8 Clarke. ($\beta=90°$.)
°								
0	0.000	0.000	0.600	0.000	0.000	0.0000	0.0000	0.0000
10	0.176	0.174	0.174	0.174	0.175	0.1747	0.1478	0.1441
20	0.364	0.352	0.342	0.348	0.349	0.3509	0.2959	0.2892
30	0.577	0.536	0.500	0.518	0.524	0.5283	0.4445	0.4361
40	0.839	0.728	0.643	0.684	0.698	0.7092	0.5941	0.5858
50	1.192	0.932	0.766	0.846	0.873	0.8938	0.7452	0.7388
60	1.732	1.154	0.866	1.000	1.047	1.0825	0.8985	0.8958
70	2.747	1.400	0.940	1.148	1.222	1.2754	1.0551	1.0567
80	5.671	1.678	0.985	1.286	1.396	1.4710	1.2169	1.2209
90	∞	2.000	1.000	1.414	1.571	1.6667	1.3863	1.3862

The relative advantages of the projections are seen still more clearly if we prepare another table (Table 6) in which, instead of taking as our unit the radius R of the sphere, we take the radius ρ_{90}, or that of the equator on the finished map; that is to say, we compare among themselves the polar maps whose limiting equatorial radii are of equal length. This table is prepared by considering the radius of 90°, as given in Table 5, as the unit for the column of figures to which it belongs. Each figure in each column is therefore divided by the value for 90° at the bottom of its column. Such a table as this is useful when there is a prescribed limit to the size of the map (such as the dimensions of the page of an atlas), and we are required to subdivide a given area into circles corresponding to the specific polar projection. An exception must be made in the case of the gnomonic projection whose radius for 90° is infinite; gnomonic charts, of course, never extend to that point, but if such a chart be limited to the polar distance 80°, we get the figures given in column 1. Table 6 shows again that Airy's development gives us slightly less distortion for the region within 30° of the equator than any other, and is by so much to be preferred for meteorological work, since in neither map is a slight distortion of the polar regions objectionable.

TABLE 6.—*Values of ρ in terms of its value at the equator (but at latitude 10° in case of gnomonic) as unity.*

North polar distance.	1 Gnomonic.	2 Stereographic.	3 Orthographic.	4 Equal surface.	5 Postel-Werner.	6 James.	7 Airy. ($\beta=90°$).	8 Clarke. ($\beta=90°$).
°								
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.031	0.087	0.174	0.123	0.111	0.104	0.107	0.104
20	0.064	0.176	0.342	0.246	0.222	0.211	0.213	0.209
30	0.102	0.268	0.500	0.368	0.333	0.317	0.321	0.315
40	0.148	0.364	0.643	0.485	0.444	0.426	0.429	0.423
50	0.211	0.466	0.766	0.600	0.556	0.537	0.538	0.533
60	0.306	0.577	0.866	0.709	0.667	0.656	0.648	0.646
70	0.485	0.700	0.940	0.814	0.778	0.766	0.761	0.762
80	1.000	0.839	0.985	0.912	0.889	0.883	0.878	0.881
90	∞	1.000	1.000	1.000	1.000	1.000	1.000	1.000

20. Polar maps with rotation in the same direction.—In the polar maps of the Northern and Southern hemispheres that I have used since 1880 (see Plates I and III), I have adhered to a principle that is generally neglected in meteorological charts, but which is all important in the mechanics of the earth's atmosphere when we come to consider its general circulation and the phenomena that depend on the diurnal rotation of the earth. The ordinary geographical maps of the Northern and Southern hemispheres are drawn as tho the observer stood over the North Pole and the South Pole, respectively, and lookt down upon the corresponding hemisphere; consequently a map of the Northern Hemisphere ordinarily represents longitude counted westward from Greenwich around the North Pole of the map as increasing in the anticyclonic or right-handed direction, while a map of the Southern Hemisphere represents the same longitudes, counted westward from Greenwich around the South Pole as increasing in the cyclonic or left-handed direction, as shown in the accompanying diagram, fig 2, Plate IV.

This method of treatment may do for descriptive geography and history and for navigators and geographers who consider only relative locations, but it is not appropriate for geophysical studies such as earthquakes. The immense inertia of the whole mass of atmosphere (revolving in one direction around the earth's axis, which we ought to call the left-handed, or positive, direction just as we do the similar direction of its annual revolution around the sun) is the most important item in meteorology, therefore we must recognize the necessity for a more rational treatment of the maps that are made for meteorological study. This is easily accomplished by drawing the polar map for the Northern Hemisphere on the plane *nn*, Plate II, as usual, viz, as seen by an observer looking down upon the earth from some point above the North Pole; then consider the earth as being transparent so that the observer, while retaining his position at or above the North Pole, looks thru the globe, as in fig. 3, Plate IV, and sees the Southern Hemisphere projected on the plane *ss* just as he had seen the Northern Hemisphere on *nn*. The two resulting maps, therefore, appear as in fig. 4, Plate IV; in both of them the longitudes circulate around the globe in the same direction as shown by arrows, L and L, while the diurnal rotation of the earth around its axis proceeds in the opposite direction as shown by the arrows R and R; the annual revolution about the sun also proceeds in this same opposite direction as shown by the arrows A and A.

By this arrangement of the maps of the Northern and Southern hemispheres, one can place the northern map above the southern with its center *n* superposed on *s*, and with a common axis of rotation so that the passage from the Northern to the Southern Hemisphere, at any point of the equator becomes continuous. In polar maps made on this system the cyclonic rotation within an area of low pressure, *x*, in the Northern Hemisphere is a positive or left-handed rotation on the map, and the so-called anticyclonic rotation around a similar area of low pressure, *y*, in the Southern Hemisphere becomes converted into a positive, a left-handed or cyclonic rotation, on the map. Thus the rules that have been formulated for ordinary usage on maps as ordinarily constructed, lose their antitheses, and the rotation about low areas is cyclonic or left-handed in both hemispheres, while the rotation about high areas is anticyclonic in both hemispheres. Any movement of the atmosphere will have a corresponding deflection toward the right on the maps of both the hemispheres alike.

If two raised maps be made according to this method, imitating the elevations and depressions of the earth's surface, one for the Northern and one for the Southern Hemisphere, respectively, and if one be placed above the other on a rotating shaft, as in fig. 5, Plate IV, and a little water be poured into the depressions on each chart, and the shaft be set in rotation, we have an approximate presentation of the action of the ocean on the globe. Experiments may thus be made with gases and liquids that shall approximately reproduce the motions of the atmosphere. By such laboratory experiments we may elucidate some of the difficulties attending the study of the general circulation of the atmosphere, since the formulas for passing from small models to the larger conditions of nature have already been given by W. von Helmholtz in his memoir on dynamic similarity.

21. Projections and models on concave surfaces.—The flat maps and models hitherto considered can serve only for a study of the motions of the lowest stratum of atmosphere, tending in general toward the equator. They must be supplemented by something better if we are to study by means of models the simultaneous motions of the upper strata which are moving in general poleward from the equator.

In the lowest stratum the general increase of temperature and humidity and the consequent diminution of density with diminution of latitude combine with the gravitational and centrifugal force to push the air toward the equator; when all this takes place on the ideal smooth sea-level surface or level surface of apparent gravity then gravity does not affect the motions except thru differences of density in masses of air of appreciable depth.

But in the upper strata the equatorial air either overflows poleward in a system of vertical circulations or overflows eastward and revolves horizontally while moving poleward in systems of circulation that soon make themselves felt at the earth's surface as areas of low pressure. In these upper strata a component of gravity is the force that overcomes the centrifugal force and other obstacles and produces the poleward flow down grade from which result the barometric gradients of our "lows" and "highs."

Hence we must devise a rotating model in which local gravitation at the laboratory shall give rise to descending poleward currents that shall simulate the overflow on the rotating globe. One way to accomplish this in a working model is to replace the flat maps by projections and models on concave curved surfaces, thus making shallow saucer-like models as in fig. 6, Plate IV. But the details of this construction belong to dynamics rather than to chartography.

THE JAMAICA HURRICANE OF OCTOBER 18-19, 1815.

By MAXWELL HALL, Esq., Government Meteorologist. Dated Chapelton, Jamaica, December 10, 1907.

This extraordinary storm, which lasted at Port Antonio for forty-eight hours, had some features resembling the hurricane of 1880. There were two centers, one of which moved slowly as it developed energy, while the other, fully developed, moved faster along its course toward the west-northwest, the usual direction. The motion of the former was abnormal; it was first toward the southwest, but when the center met the Blue Mountain Range south of Port Antonio, it stopt and even recoiled, and then advanced slowly again toward the southwest and Kingston.

Dr. W. Arnold has given a detailed account of the storm, as experienced at Port Antonio, in Vol. II of the Jamaica Physical Journal; he took great pains with the varying directions of the wind, and tabulated them at the end of his account so that there should be no mistake, and by means of a brief account of what occurred in Kingston, as given in the Royal Gazette, it is possible to make a short study of this storm. The small provisional maps attached to this article will be found useful.

Extract from the Royal Gazette.[1]

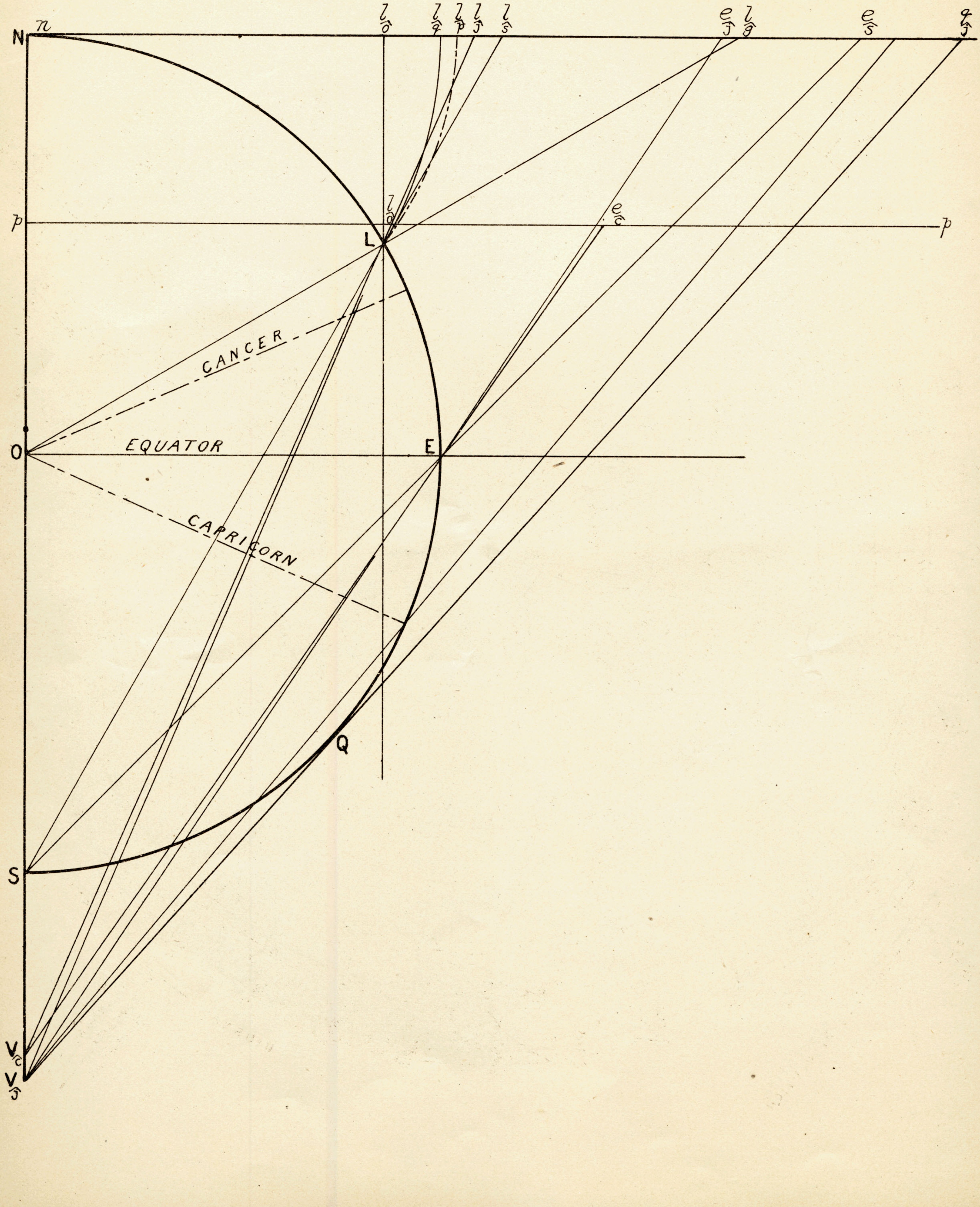
KINGSTON, *Oct. 21st.*

SEVERE STORM.

On Tuesday, Oct. 17th, during the afternoon a heavy fall of rain set in, with a fair prospect of good October seasons; but about two o'clock on Wednesday morning it began to blow extremely hard from the eastward, from whence it changed to the SE; and on the following day it shifted to different quarter of the compass, from N to NW and W, and thence to N,

[1] From Saturday October 14, to Saturday October 21, 1815.


ANNUAL ORBITAL REVOLUTION
ROTATION
DIURNAL
ANNUAL ORBITAL
DIURNAL
LIBRA
MARCH 21.
180°
REVOLUTION
ROTATION
N
DIURNAL
ANNUAL
SUN
ARIES
ROTATION
ORBITAL REVOLUTION
DIURNAL
ANNUAL ORBITAL REVOLUTION
ROTATION


N
n
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L
O
EQUATOR
E
CANCER
CAPRICORN
Q
S

Polar Map of the Southern Hemisphere.

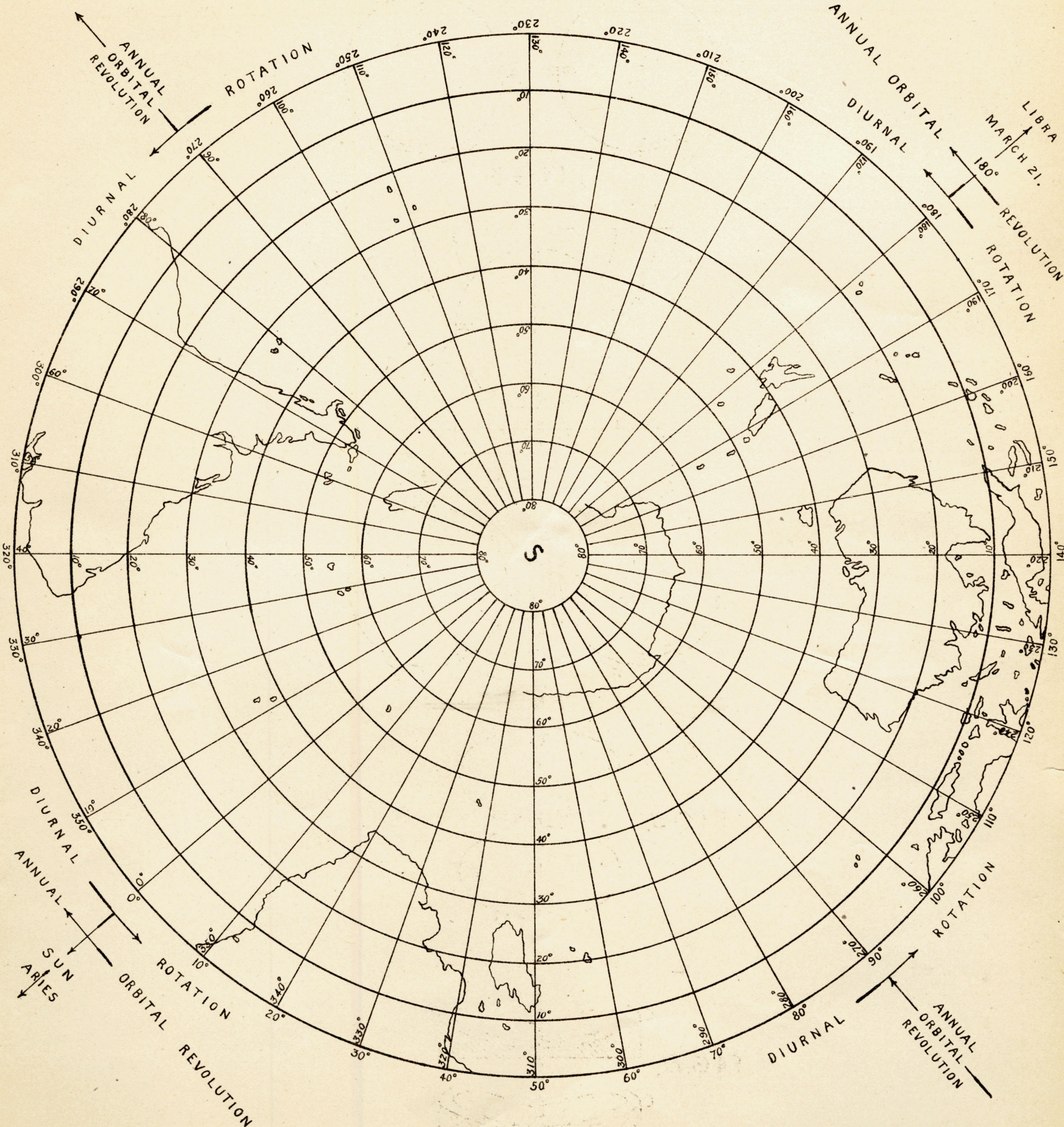

FIG. 2.

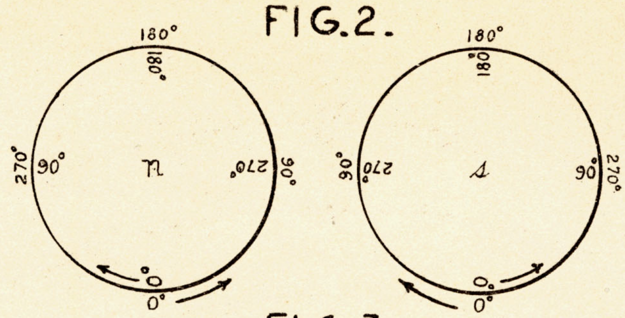

FIG. 3.

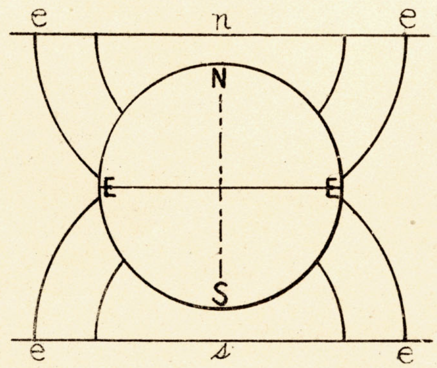

FIG. 4.

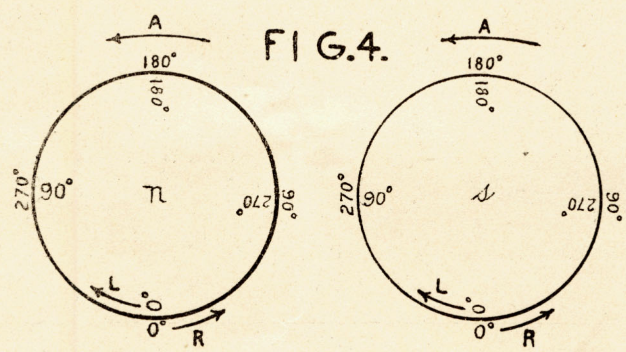

FIG. 5.

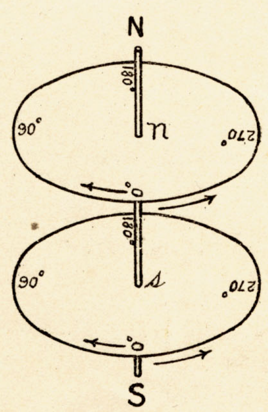

FIG. 6.